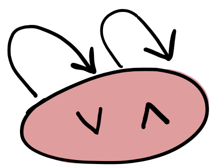
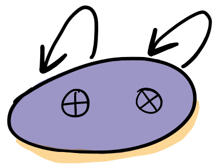
The Fine-grained Complexity of XNFA and NIFA acceptance





DMITRY CHISTIKOV[1] RADOSŁAW PIÓRKOWSKI[1]

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1 = University of Warwick, UK. 2 = Stellenbosch University, South Africa

In this talk:

I NFA and XNFA acceptance

II Degrees of ambiguity

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II Degrees of ambiguity

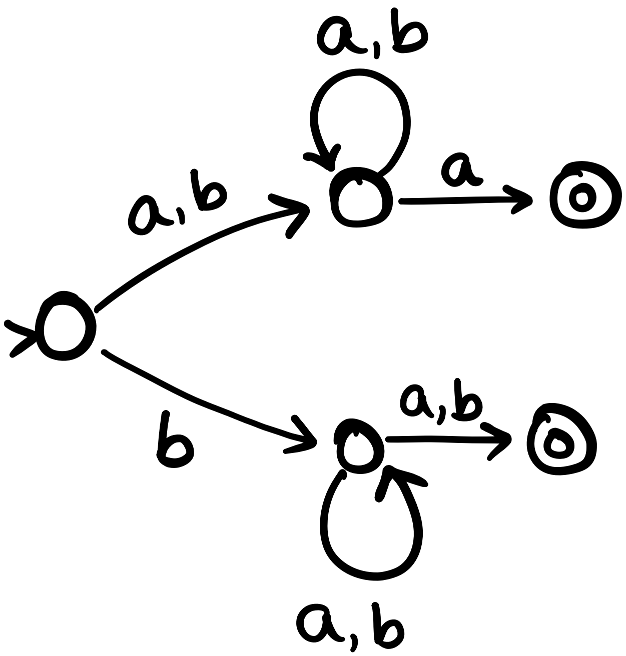
Our Results

III Unambiguous (X)NFA are easy

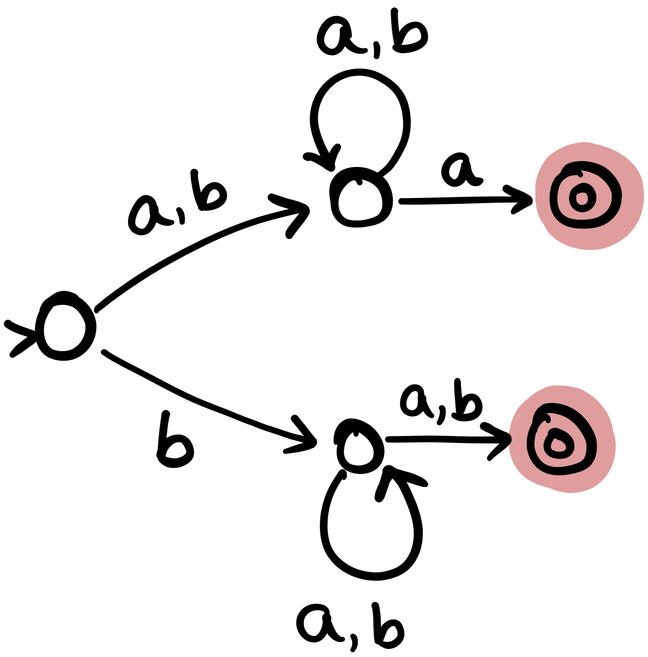
IV Polynomially ambiguous NFA Acc reduce to XNFA Acc

NFA

(nondeterministic finite automata)



NFA
(nondeterministic finite automata)



Positive number of runs

NFA

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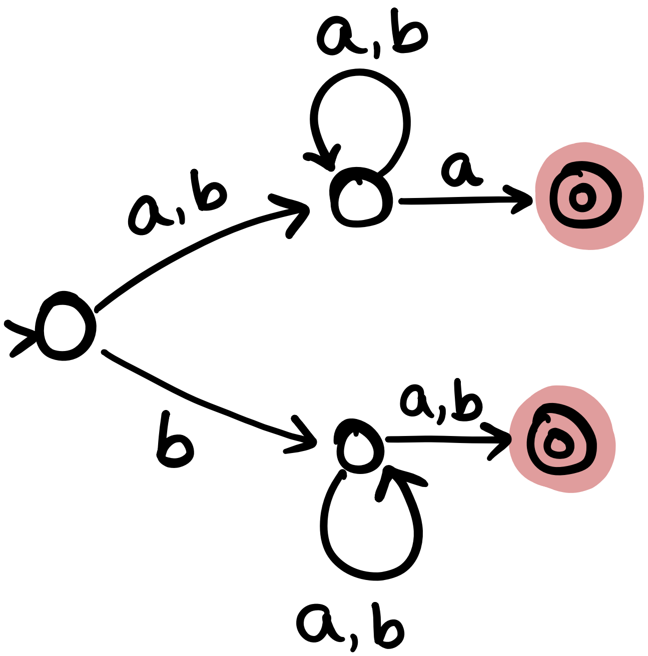
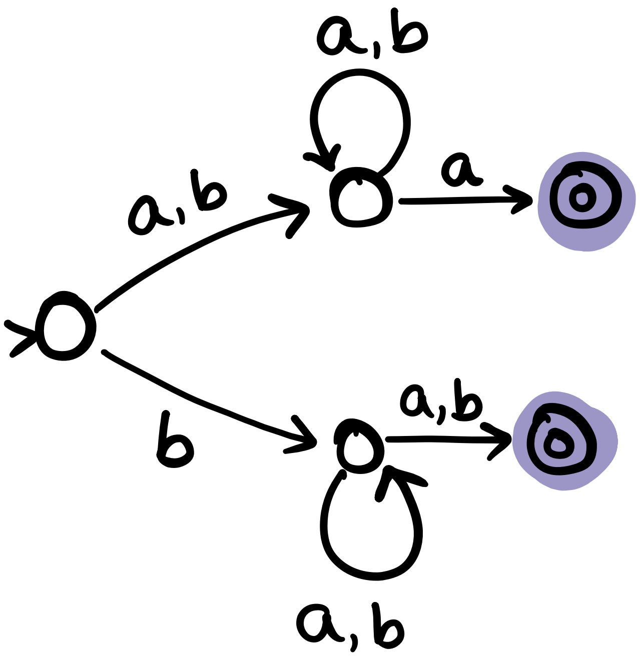


Positive number of runs

$L(A) = \{ w \mid (|w| \geq 2) \text{ and } (\text{begins with } b \text{ or ends in } a) \}$

XNFA
(symmetric difference automata)



Odd number of runs

XNFA
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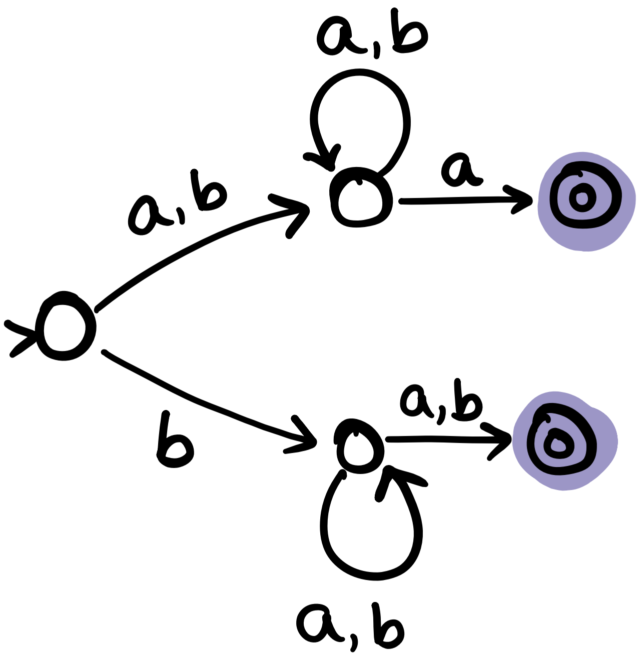
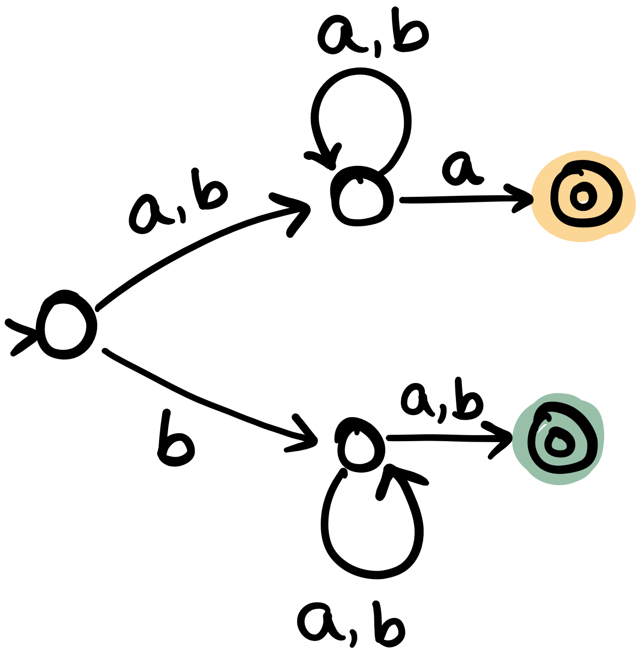


Odd number of runs

$L(A) = \{ w \mid (|w| \geq 2)$ and 1st letter = last letter$\}$



NFA

$L = L_{\circledcirc} \cup L_{\circledcirc}$

XNFA

$L = L_{\circledcirc} \oplus L_{\circledcirc}$

NFA

Recognise regular languages

Exponentially more succinct than DFA

XNFA

Recognise regular languages

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NFA

Recognise regular languages

Exponentially more succinct than DFA

Weighted automata over:

$\mathbb{B} = \{0, 1, \vee, \wedge\}$

$0 \vee 0 = 0$ $\quad 1 \vee 0 = 0 \vee 1 = 1$

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Equivalence is PSPACE-complete.

XNFA

Recognise regular languages

Exponentially more succinct than DFA

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Equivalence is $O(n^3)$-time

The Acceptance Problem

Input: A word w, an n state m transition automaton A

Output: Is $w \in L(A)$?

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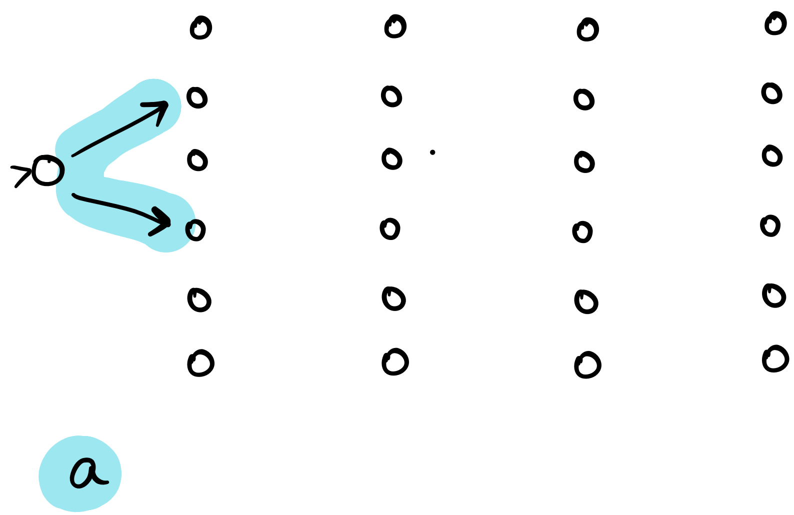
NFA Acceptance | XNFA Acceptance



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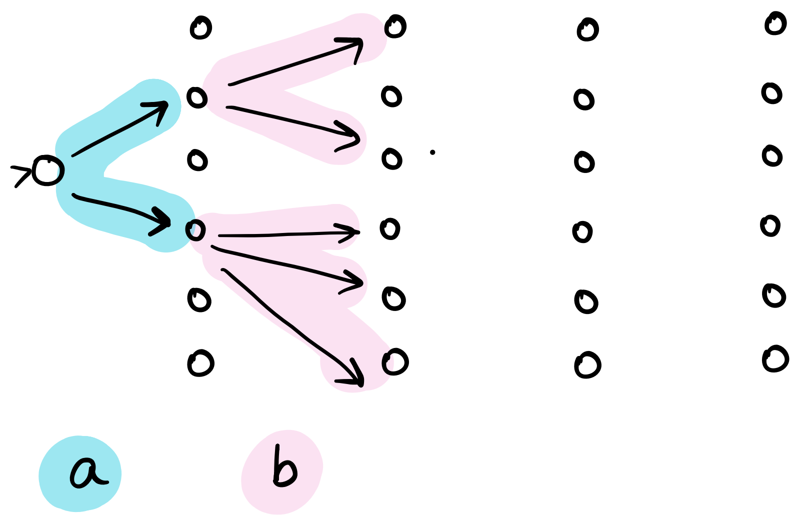

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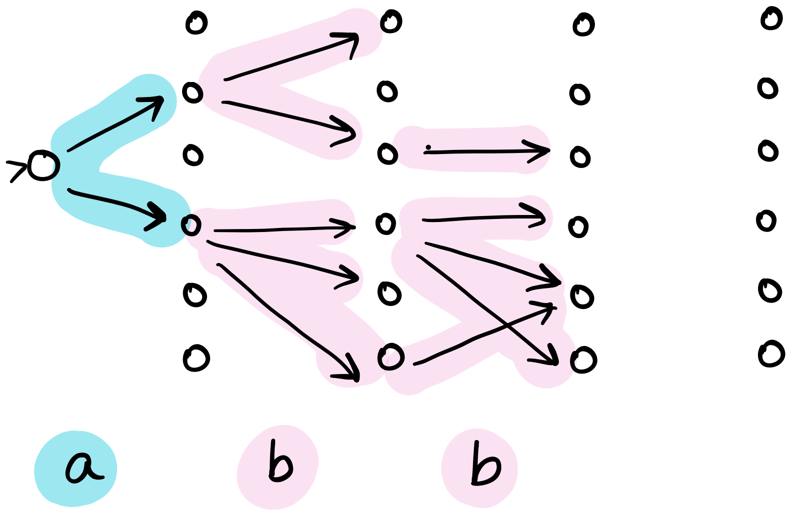

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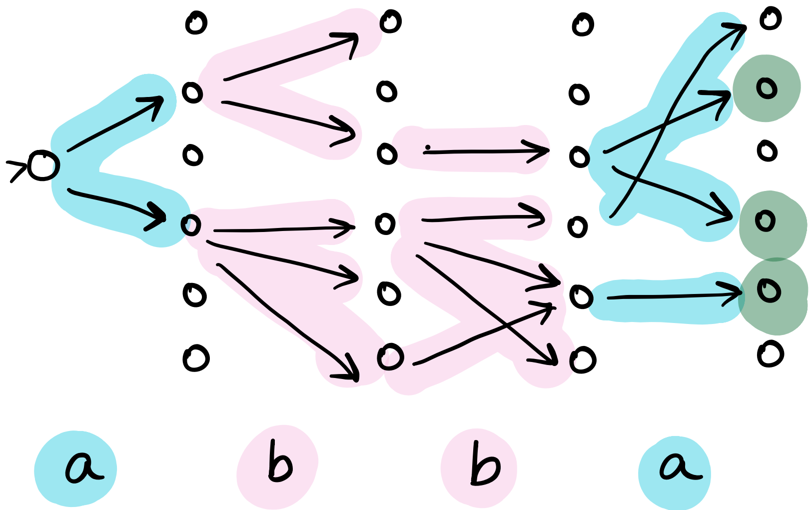

$O(m\,|w|)$-time

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No $O((m|w|)^{0.999\ldots})$-time algorithm known.

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NFA Acceptance Hypothesis
[Bringmann, Grønlund, Künnemann & Larsen, TCS 2024]

Does the field structure of $\mathbb{F}_2$ make it easier?

Today's Question

Are NFA Acceptance and XNFA Acceptance related in a fine-grained sense?

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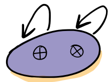
Analogy: $\mathrm{NL} \subseteq \oplus\mathrm{L}$?

$\mathrm{NL} \supseteq \oplus\mathrm{L}$?

[Immerman and Landau, '95] [Damm, '90]

In this talk:

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Tool: Degree of ambiguity.

[Weber & Seidl, TCS '91]

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III Exponentially Ambiguous : $\exists\, c \in \mathbb{R}, \forall w \in \Sigma^*,$

$$\text{runs}(w) > c^{|w|}$$

Tool: Degree of ambiguity

[Weber & Seidl, TCS '91]

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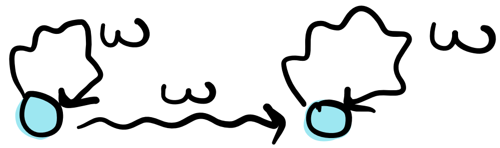
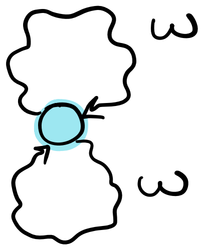
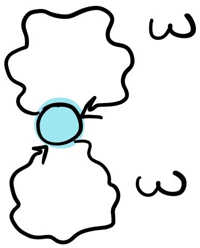


II Polynomial Ambiguous :



III Exponentially Ambiguous : ✓



Tool: Degree of ambiguity

[Weber & Seidl, TCS '91]

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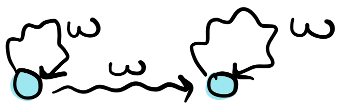
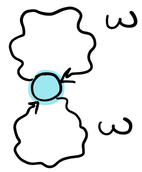


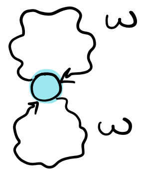
II Polynomial Ambiguous :



O(n^3) and O(n^2)-time decidable.

[Drabik, Dürr, Frei, Mazowiecki & Węgrzycki]

III Exponentially Ambiguous :



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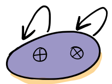
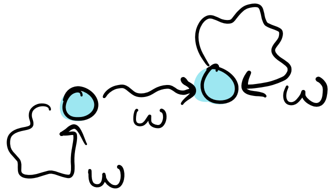


I NFA and XNFA acceptance



II Degrees of ambiguity

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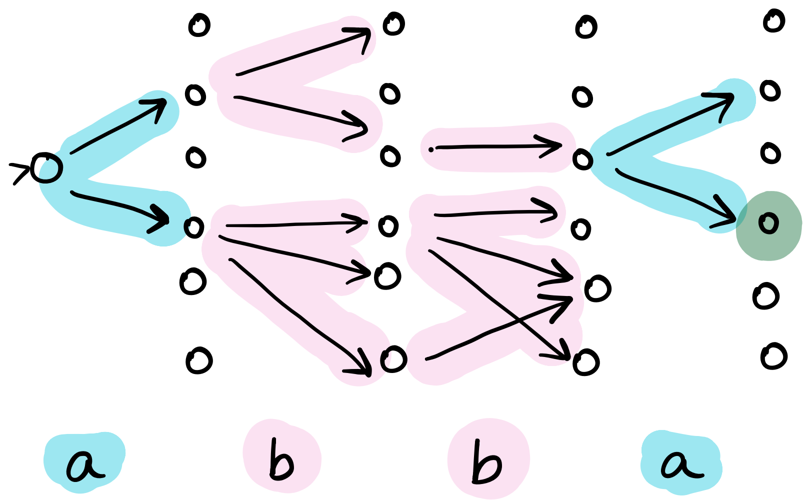
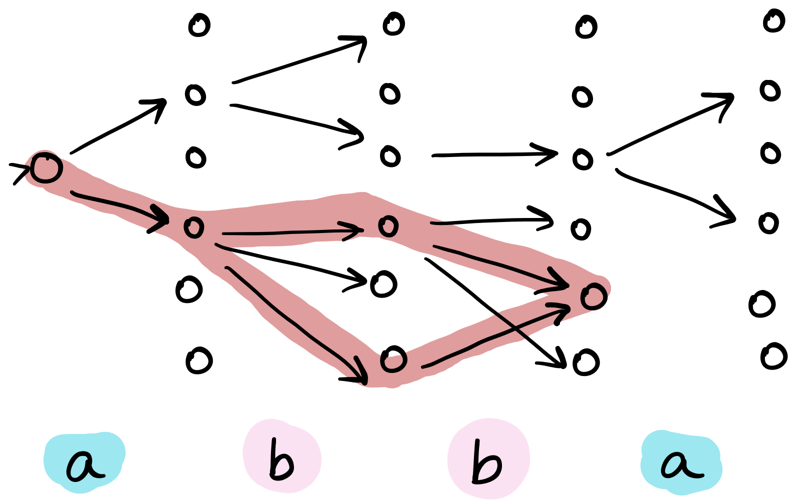
Unambiguous Acceptance



Step one: Trim A — O(m+n)-time

Unambiguous Acceptance



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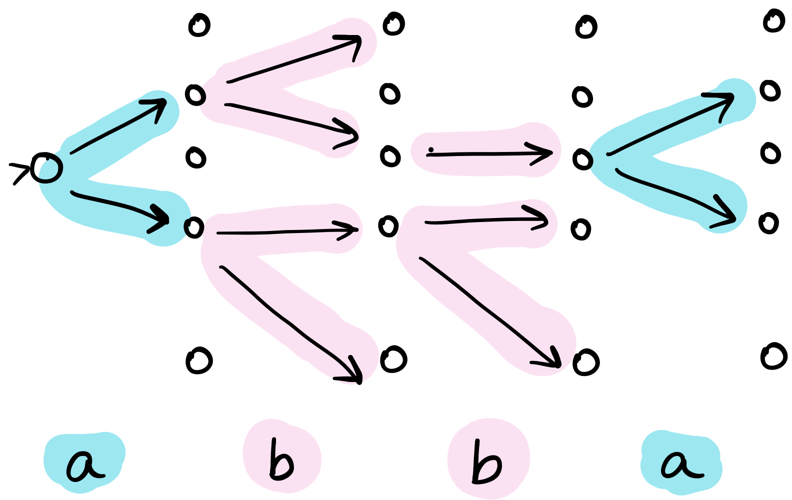
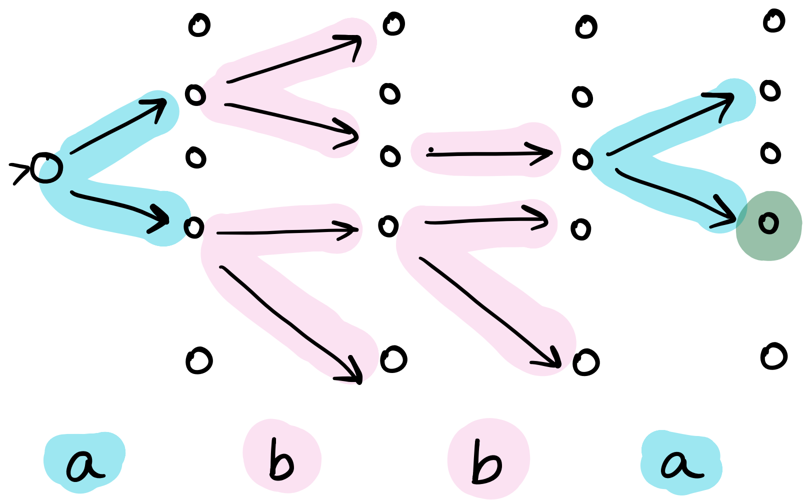


$O(m + n \cdot |w|)$-time

$n = \#$ of states
$m = \#$ of transitions

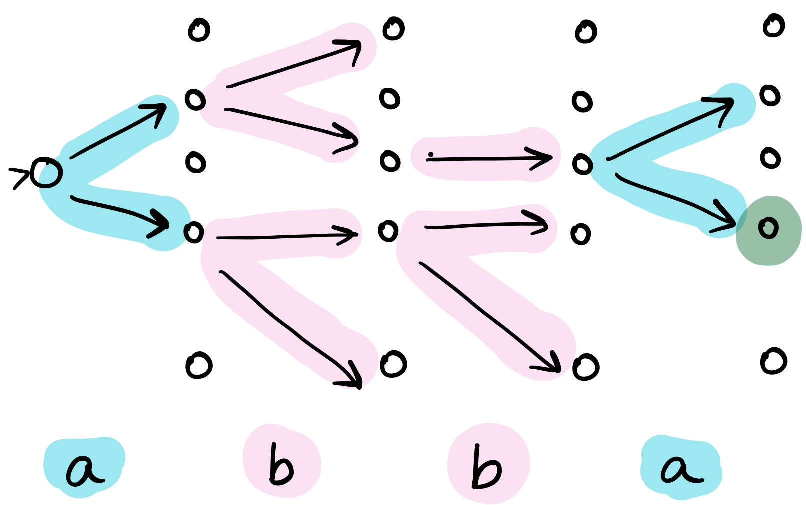
Finitely ambiguous Acceptance



$O(m + k \cdot n \cdot |w|)$-time if runs$(w) < k$

n = # of states
m = # of transitions

Finitely ambiguous Acceptance



[Kiefer & Ryzhikov, STACS '25]

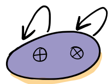
$O(m + k \cdot n \cdot |w|)$-time

Q. 0-1 matrix multiplication in $O(n^2)$ time?

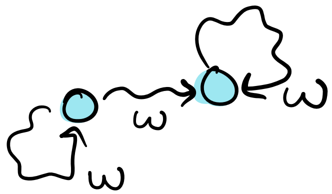
n = # of states
m = # of transitions

Ans. Yes!

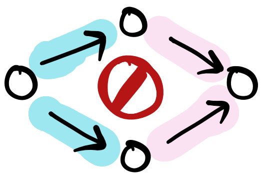
In this talk:



I NFA and XNFA acceptance



II Degrees of ambiguity



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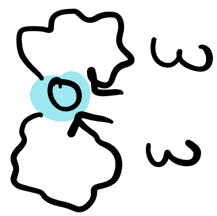
Polynomially Ambiguous NFA

There is a polynomial $p: \mathbb{N} \to \mathbb{N}$ such that every word w has at most $p(|w|)$ accepting runs. $\iff$ no



[Weber & Seidl, TCS '91]

<u>Note</u>: All known hard instances of NFA Acceptance are polynomially ambiguous.

[Bringmann et. al]

[Potechin & Shallit, IPL '20]

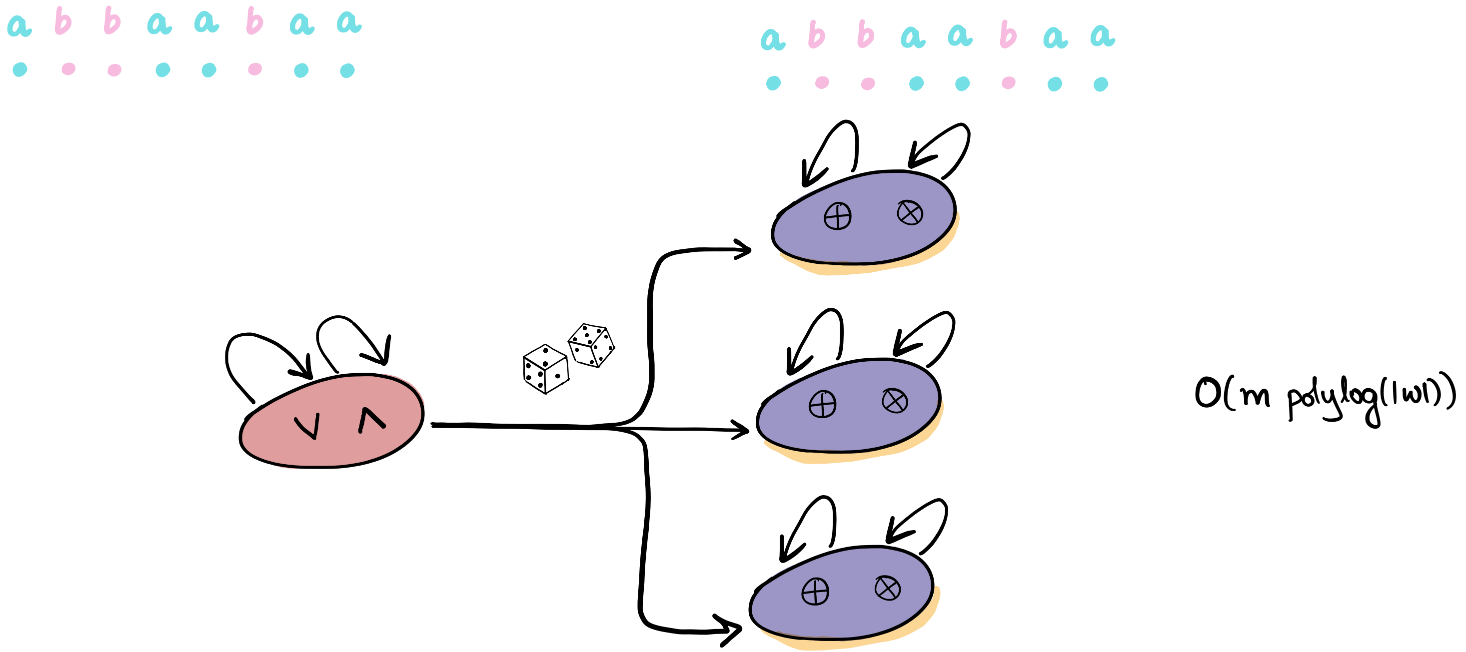
Polynomially Ambiguous NFA Acc $\leq$ XNFA Acc

a b b a a b a a

a b b a a b a a

O(m) size,
O(m) time constructible.

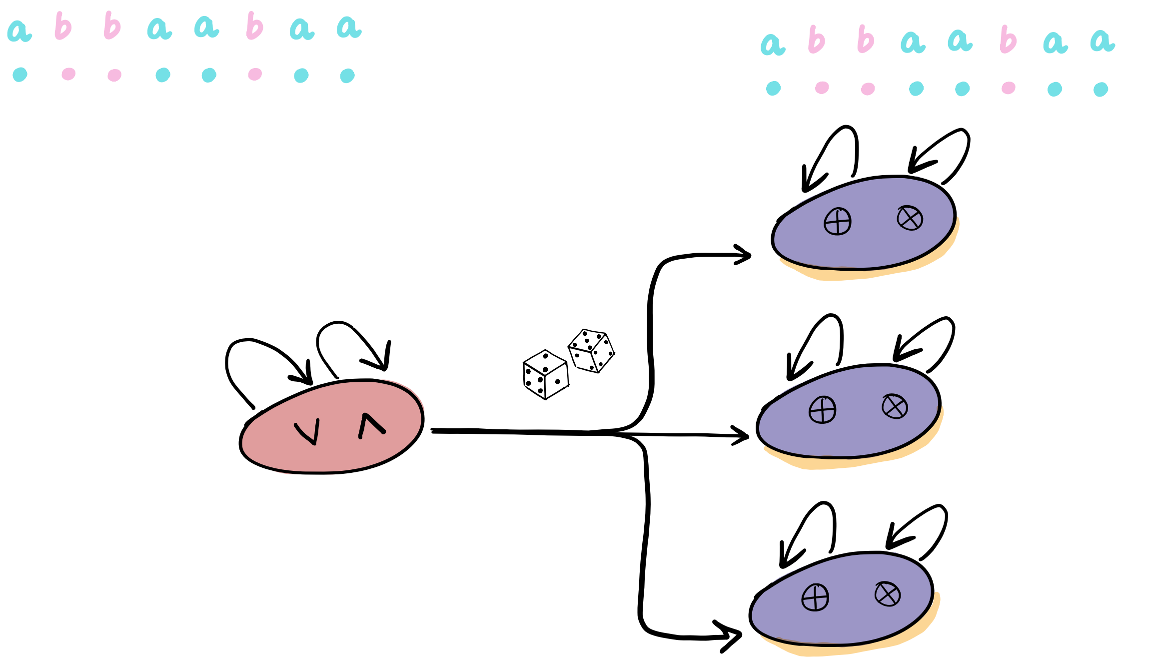
Polynomially Ambiguous NFA Acc $\leq$ XNFA Acc



$O(m \operatorname{polylog}(|w|))$-time constructible,

$O(m \log(|w|))$-size.

Polynomially Ambiguous NFA Acc $\leq$ XNFA Acc



$O(m \, \mathrm{polylog}(|w|))$

$$w \notin \mathcal{L}(A) \implies w \notin \mathcal{L}(A_i)$$

$$w \in \mathcal{L}(A) \implies p > \frac{1}{2},\ \exists i \ w \in \mathcal{L}(A_i)$$

XNFA-Acc is poly. amb.
NFA-Acc hard.

Thm. If there is an $O((m|w|)^{0.999...})$-time algorithm for XNFA Acceptance, the NFA Acceptance hypothesis is false for polynomially ambiguous NFA.

Poly. amb. NFA $\leq$ XNFA :

Lemma 1 : If A is polynomially ambiguous, no two accepting runs on w are permutations of each other.

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Schwartz-Zippel lemma: Given a d-degree polynomial $q \in F\langle x_1, \ldots, x_\ell \rangle$ over a field F, if q is syntactically nonzero, q evaluates to zero over elements of F with probability $\leq \dfrac{d}{|F|}$

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Lemma 2: We can use k XNFAs to simulate a weighted automaton over $GF(2^k)$.

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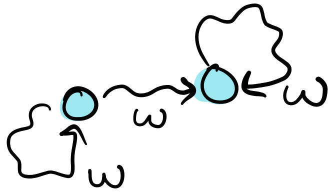

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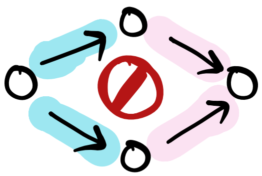

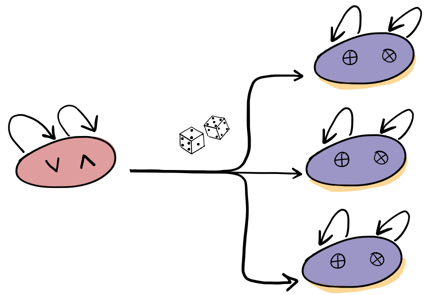

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